

# Optimum Lossy Broadband Matching Networks for Resonant Antennas

*This analysis determines the optimum component values for obtaining a broadband match between a transmission line and an antenna, using a transformer and a resonator.*

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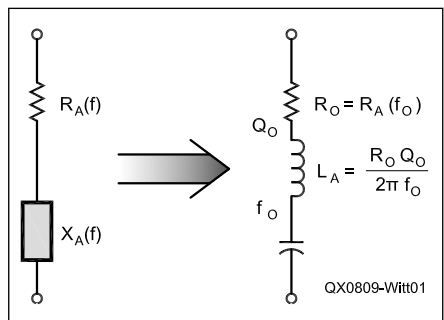
This article was originally published in *RF Design* in 1990 and is reprinted here by permission of the publisher.<sup>1</sup> It is the primary reference for work AI1H has done on broadband matching documented in ARRL publications.<sup>2, 3, 4, 5, 6, 7, 8, 9</sup> It is the only reference that contains the derivation of the pertinent formulas, and it ties it to the classic work on the subject by Fano.<sup>10</sup>

Examples of practical resonant antennas are the quarter-wave monopole, (n/2)-wavelength dipoles, where n is an odd positive integer, and the full-wave loop. Another example is a short non-resonant antenna, which has been made resonant by the addition of a reactive element. The resonant antenna has inherent broadband radiating properties, but its match bandwidth falls short of meeting requirements in many applications.

This paper addresses the problem of optimizing the design of a fixed matching network located between the transmission line and the resonant antenna. The matching network contains a transformer and a resonator. The analysis differs from previous results in that it yields explicit formulas, which provide maximum match bandwidth and which account for the incidental losses that are inherent in most practical matching networks.<sup>10, 11, 12</sup> The goal of the optimization presented herein is to maximize the match

bandwidth for a given maximum standing wave ratio (SWR) over the operating band.

In his classic work, Fano<sup>10</sup> addressed the same problem in a very general sense, but treated only the lossless case. Fano's generality included a wide class of load impedances and high order matching networks. He and others have observed that large match bandwidth improvements are possible with very simple matching networks.<sup>11, 13, 14</sup> This paper treats such a case. A recent examination of the same single resonator matching network structure was reported by Hansen.<sup>12</sup> He recognized the importance of accounting for matching network loss, but did not include it explicitly in his evaluation. The value of the results described herein is that concise design formulas, which account for the losses in the matching network, are provided.



**Figure 1 — Dipole approximate equivalent circuit used in the analysis.  $R_A(f)$  includes both the radiation resistance and antenna losses.**

## Antenna Impedance

The analysis that follows applies to antennas whose impedance near resonance may be approximated by a series RLC circuit as shown in Figure 1. The use of this approximation makes the analysis tractable, and, as will be seen later, is accurate enough to provide useful design information. The results apply to the dual case as well, however this case will not be covered here.

It will be assumed that the real part of the antenna impedance,  $R_A(f)$ , does not vary much over the operating band. It is set equal to the value of the real part of antenna impedance at resonance,  $R_O$ . The antenna driving point impedance is thus established by three parameters: the resonant frequency,  $F_O$ , the real part of the antenna impedance at resonance,  $R_O$ , and the  $Q$  of the antenna at resonance,  $Q_O$ :

$$Q_O = \frac{F_O}{2R_O} \cdot \left. \frac{dX_A(f)}{df} \right|_{f=F_O} = \frac{2\pi F_O L_A}{R_O} \quad [\text{Eq 1}]$$

where  $L_A$  = antenna inductance.

## Transformer Matching

Before treating the transformer/resonator matching case, it is helpful to consider the simplest form of matching network, which consists of a transformer whose bandwidth is large compared with the bandwidth of the resonant antenna.<sup>5</sup> The topology, which assumes that the transformer losses are negligible, is shown in Figure 2A. Maximum bandwidth is not achieved when a perfect match (SWR = 1:1) exists at resonance. In Figure 3, the intentional mismatch at resonance is seen. That

<sup>1</sup>Notes appear on page 40.

mismatch is achieved by driving the antenna with a generator whose impedance is greater than  $R_o$ , causing the SWR to equal  $S_L$  at resonance. A generator impedance lower than  $R_o$  will not yield optimum matching.

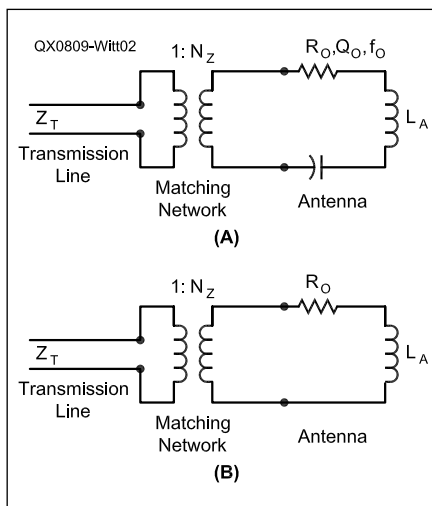
The analysis is facilitated by transforming the band-pass network of Figure 2A to the low-pass network of Figure 2B. The SWR versus frequency characteristic of Figure 3 shows the relationship between the band-pass and low-pass SWR characteristics. The midband frequency,  $F_o$ , is the geometric mean of the band edge frequencies,  $F_L$  and  $F_H$ . Of interest is the bandwidth,  $BW$ , at a particular SWR,  $S_M$ . The reflection coefficient looking from the line into the matching network is given by:

$$\rho(f) = \frac{R_o + j2\pi fL_A - Z_T N_Z}{R_o + j2\pi fL_A + Z_T N_Z} = \frac{(1 - S_L) + j\left(\frac{f}{F_o}\right) Q_o}{(1 + S_L) + j\left(\frac{f}{F_o}\right) Q_o} \quad [\text{Eq 2}]$$

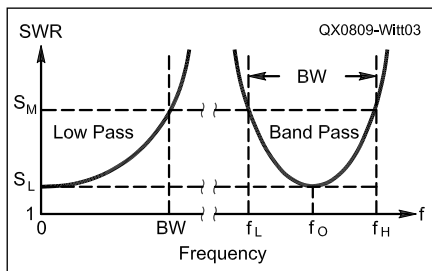
where:

$Z_T$  = transmission line characteristic impedance (ohms)

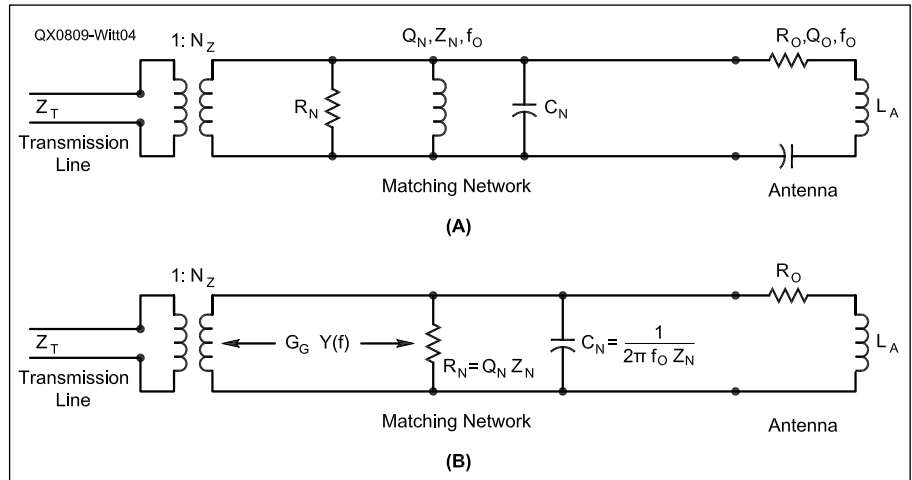
$N_Z$  = transformer impedance ratio.



**Figure 2 — Transformer matching. Part A shows the equivalent circuit. Part B shows the circuit after the band-pass to low-pass transformation.**



**Figure 3 — SWR versus frequency for optimum transformer matching.**



**Figure 4 — Transformer/resonator matching. Part A shows the equivalent circuit and Part B shows the results after the band-pass to low-pass transformation.**

Substituting  $f = BW$  in Equation 2 and using

$$|\rho(BW)| = \frac{S_M - 1}{S_M + 1} \quad [\text{Eq 3}]$$

yields

$$BW = \frac{F_o}{Q_o} \left[ \left( S_M + \frac{1}{S_M} \right) S_L - S_L^2 - 1 \right]^{\frac{1}{2}} \quad [\text{Eq 4}]$$

It is useful to define the normalized bandwidth,  $B_N$ , to be the product of the fractional bandwidth and the antenna  $Q$ :

$$B_N = \frac{BW}{F_o} \cdot Q_o \quad [\text{Eq 5}]$$

Also, the normalized reference bandwidth,  $B_{Nref}$  is defined to be the normalized bandwidth for the case when the antenna is perfectly matched at resonance. Using Equation 4 for  $S_L = 1$  yields:

$$B_{Nref} = \frac{S_M - 1}{\sqrt{S_M}} \quad [\text{Eq 6}]$$

By setting

$$\frac{dB_N(S_L)}{dS_L} = 0$$

the value of  $S_L, S_{Lopt}$ , which gives the maximum bandwidth, may be determined:

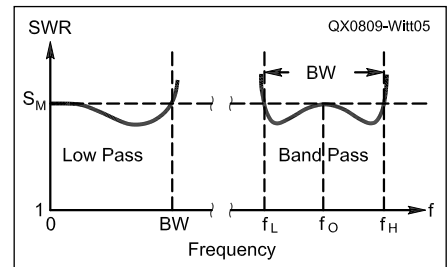
$$S_{Lopt} = \frac{1}{2} \left( S_M + \frac{1}{S_M} \right) \quad [\text{Eq 7}]$$

Substitution yields the maximum normalized bandwidth,  $B_{Nmax}$ :

$$B_{Nmax} = \frac{1}{2} \left( S_M - \frac{1}{S_M} \right) \quad [\text{Eq 8}]$$

The required transformer impedance ratio is:

$$N_Z = \frac{R_o}{2Z_T} \left( S_M + \frac{1}{S_M} \right) \quad [\text{Eq 9}]$$



**Figure 5 — SWR versus frequency for optimum transformer/resonator matching.**

The improvement in match bandwidth with transformer matching is modest. If  $S_M = 2:1$ , for example, by deliberately mismatching at resonance so that  $S_L = S_{Lopt} = 1.25:1$ , the 2:1 SWR bandwidth is improved by only about 6%.

### Transformer/Resonator Matching

In dramatic contrast to transformer matching, a matching network using a transformer in combination with a resonant circuit yields a significant improvement in match bandwidth. It is important, however, to properly account for losses in the matching network since these losses will influence the values of the design parameters in most practical applications. Fortunately, though somewhat tedious to derive, the results may be expressed as explicit formulas.

The antenna system for this case is shown in Figure 4A. In the matching network, the transformer is used to deliberately raise the generator impedance seen by the antenna at midband to a high but acceptable value. The resonant circuit, which has the same resonant frequency as the antenna, is used to partially compensate for the reactance of the antenna for frequencies away from resonance. All of the losses in the matching network are rep-

resented by the  $Q$  of the matching network resonator,  $Q_N$ . In addition to the transmission line and antenna parameters,  $Q_N$  is assumed to be known.

The same analysis approach used in the transformer matching case is used here. Refer to Figure 5. The parameters of the matching network are chosen so that the SWR at mid-band and at the band edges equals  $S_M$ . The optimization process involves determining the transformer impedance ratio,  $N_Z$ , and the matching resonator impedance level,  $Z_N$ , so that maximum bandwidth is achieved.  $Z_N$  is the impedance of the resonator inductor or capacitor at the antenna resonant frequency. [In subsequent writing, including Chapter 9 of The ARRL Antenna Book, the author changed the notation for the matching network impedance level (the reactance of the matching inductor or matching capacitor at antenna resonance) from  $Z_N$  to  $X_{NO}$ . We did not make that change in this article.—Ed.]

Referring to the low-pass equivalent circuit of Figure 4B, at dc:

$$N_Z = \frac{S_M Q_N Z_N R_O}{Z_T Q_N Z_N + R_O} \quad [\text{Eq 10}]$$

Implicit in this equation is the assumption that the optimum condition occurs when the SWR is equal to  $S_M$  at the center of the band as well as at the band edges. The author has been able to prove this for the lossless case and has been unable to disprove it for the lossy case.

The matching network loss,  $L_{MN}$ , is defined as the ratio, expressed in decibels, of the total power delivered by the transmission line to the power delivered to the antenna load. It is given by:

$$L_{MN} = 10 \log \frac{G_A + \left( \frac{1}{Q_N Z_N} \right)}{G_A} \quad [\text{Eq 11}]$$

where  $G_A$  is the antenna conductance, which is given by:

$$G_A = \frac{1}{R_O \left\{ 1 + \left[ \left( \frac{f}{F_O} \right) Q_O \right]^2 \right\}} \quad [\text{Eq 12}]$$

Thus:

$$L_{MN} = 10 \log \left( 1 + \frac{R_O}{Q_N Z_N} \left\{ 1 + \left[ \left( \frac{f}{F_O} \right) Q_O \right]^2 \right\} \right) \quad [\text{Eq 13}]$$

In the band-pass domain, at midband:

$$L_{MNO} = 10 \log \left( 1 + \frac{R_O}{Q_N Z_N} \right) \quad [\text{Eq 14}]$$

and at the band edges:

$$L_{MNE} = 10 \log \left[ 1 + \frac{R_O}{Q_N Z_N} (1 + B_N^2) \right] \quad [\text{Eq 15}]$$

For any application, the matching network loss is highest at the edges of the band, so  $L_{MNE}$  will usually be the most important loss parameter.

The expressions for transformer impedance ratio and matching network loss pertain to the topology of Figure 4A and are very general. They apply for any matching network impedance level and normalized bandwidth. Some expressions for impedance level, normalized bandwidth and band edge loss for cases of particular interest are presented next.

### Maximum Bandwidth

It is possible to write an explicit expression that relates  $Z_N$  and  $B_N$ . This is done by first determining the generator conductance,  $G_G$ , necessary to achieve  $\text{SWR} = S_M$  at resonance.

$$G_G = \frac{R_O + Q_N Z_N}{S_M R_O Q_N Z_N} \quad [\text{Eq 16}]$$

The admittance,  $Y(f)$ , facing the generator is given by:

$$Y(f) = \frac{1}{Q_N Z_N} + j \frac{f}{F_O Z_N} + \frac{1}{R_O \left[ 1 + j \left( \frac{f}{F_O} \right) Q_O \right]} \quad [\text{Eq 17}]$$

The magnitude of the reflection coefficient is then:

$$|\rho(f)| = \frac{|Y(f) - G_G|}{|Y(f) + G_G|} = \frac{\text{SWR} - 1}{\text{SWR} + 1} \quad [\text{Eq 18}]$$

At the band edges,  $f = \text{BW}$  and  $\text{SWR} = S_M$ . With these substitutions, the general expressions for  $B_N$  and  $Z_N$  may be derived:

$$B_N = \left[ 2 \left( S_M + \Delta \right) \frac{Z_N Q_O}{R_O S_M} - \left( \frac{Z_N Q_O}{R_O S_M} \right)^2 - 1 \right]^{\frac{1}{2}} \quad [\text{Eq 19}]$$

and

$$Z_N = \frac{R_O S_M}{Q_O} \left\{ S_M + \Delta \pm \left[ \left( S_M + \Delta \right)^2 - 1 - B_N^2 \right]^{\frac{1}{2}} \right\} \quad [\text{Eq 20}]$$

where:

$$\Delta = \frac{Q_O}{2 Q_N} \left( S_M - \frac{1}{S_M} \right)$$

Note that for the lossless matching network case,  $\Delta = 0$ . The optimum impedance level,  $Z_{Nopt}$ , which yields the maximum bandwidth, is determined by setting

$$\frac{dB_N}{dZ_N} = 0$$

and solving for  $Z_N$ .

$$Z_{Nopt} = \frac{R_O S_M}{Q_O} (S_M + \Delta) \quad [\text{Eq 21}]$$

The maximum normalized bandwidth,  $B_{NMax}$  is:

$$B_{Nmax} = \left[ \left( S_M + \Delta \right)^2 - 1 \right]^{\frac{1}{2}} \quad [\text{Eq 22}]$$

The large bandwidth enhancement obtained by using the transformer/resonator matching network is seen from the following example: For the lossless case and  $S_M = 2:1$ ,

$$B_{Nref} = \frac{1}{\sqrt{2}} \quad \text{and} \quad B_{Nmax} = \sqrt{3}.$$

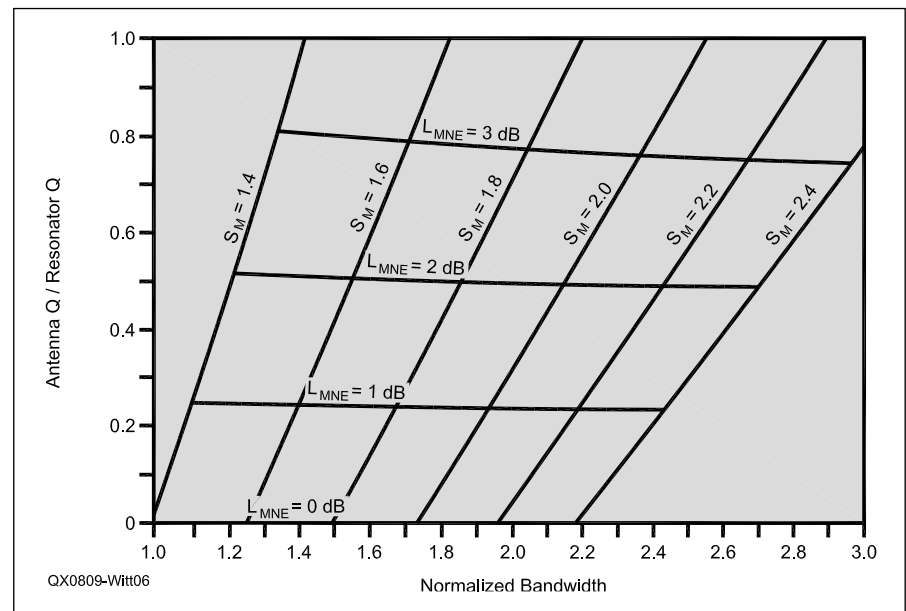


Figure 6 — Tradeoff between bandwidth, match quality and matching network loss.

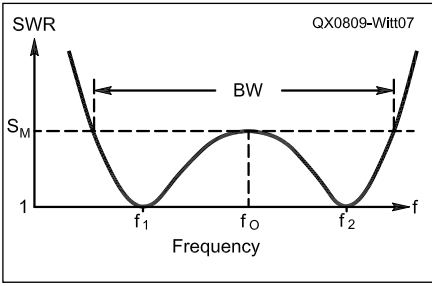


Figure 7 — Perfect match at two frequencies.

Hence, the bandwidth is increased by a factor of 2.45 over the case of a dipole matched at resonance. It is clear from Equation 22 that the bandwidth is increased even further when a lossy matching network is used.

The matching network loss at the band edges,  $L_{MNE}$ , is given by:

$$L_{MNE} = 10 \log \left\{ 1 + \frac{Q_o}{Q_N} \left[ \frac{Q_o}{2Q_N} \left( 1 - \frac{1}{S_M^2} \right) + 1 \right] \right\} \quad [\text{Eq 23}]$$

Notice the weak dependence on  $S_M$ . For cases when the matching network resonator  $Q$  is at least an order of magnitude greater than the antenna  $Q$ , the band edge loss simplifies to:

$$L_{MNE} = 10 \log \left( 1 + \frac{Q_o}{Q_N} \right) \quad [\text{Eq 24}]$$

The above analysis provides the basis for a graphical representation of the relationship between normalized bandwidth, matching network loss,

$$\frac{Q_o}{Q_N}$$

and  $S_M$ . See Figure 6. For a particular application, where antenna  $Q$  and matching network resonator  $Q$  are given, this figure is very useful for quickly determining the tradeoff among match quality, bandwidth and matching network loss.

In most practical situations, the operating band over which matching is desired is given. In those cases, one wishes to know the best match achievable,  $S_{Mmin}$ , for a given normalized bandwidth,  $B_N$ . Solving Equation 22 for  $S_M$  yields:

$$S_{Mmin} = \frac{B_N^2 + 1 + \left\{ B_N^2 + 1 + \left( \frac{2Q_o}{Q_N} \right) \left[ 1 + \left( \frac{Q_o}{2Q_N} \right) \right] \right\}^{\frac{1}{2}}}{2 \left[ 1 + \left( \frac{Q_o}{2Q_N} \right) \right]} \quad [\text{Eq 25}]$$

#### Minimizing Matching Network Loss

For situations where either the maximum possible bandwidth or minimum possible SWR is not required, Equation 20 may be used to determine the necessary value of  $Z_N$ . Notice that for values of normalized bandwidth less than  $B_{Nmax}$  there are two values of

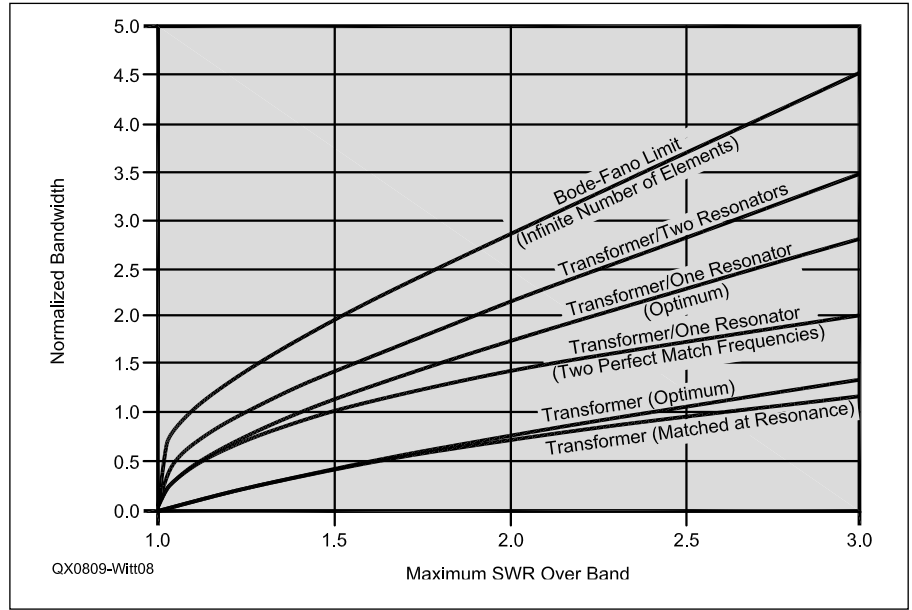


Figure 8 — Normalized bandwidth versus maximum SWR over the band for the lossless matching network case.

$Z_N$ . The larger one is usually selected in order to minimize the matching network loss. An example later will show the potentially large impact of making the proper selection.

#### Perfect Matching at Two Frequencies

It is possible to find a value of  $Z_N$  that provides a perfect match at two frequencies, as seen in Figure 7.<sup>2</sup> For this case,

$$Z_N = \frac{R_o}{Q_o} \left[ S_M + \frac{Q_o}{Q_N} (S_M - 1) \right] \quad [\text{Eq 26}]$$

and

$$B_N = (S_M - 1)^{\frac{1}{2}} \left\{ 2 + \frac{Q_o}{Q_N} \left[ 2 + \left( 1 + \frac{Q_o}{Q_N} \right) \left( 1 - \frac{1}{S_M} \right) \right] \right\}^{\frac{1}{2}} \quad [\text{Eq 27}]$$

This case may satisfy a special need, but it yields smaller bandwidth and more loss than the case presented in the previous section. For the case when  $S_M = 2:1$ , the achievable bandwidth is about 18% smaller than the maximum attainable with the same topology. This result is analogous to the transformer matching case, where obtaining perfect match at a frequency within the band does not yield maximum bandwidth.

The perfect match frequencies are given by:

$$F_1 = (F_o^2 + F_M^2)^{\frac{1}{2}} - F_M \quad [\text{Eq 28}]$$

and

$$F_2 = \frac{F_o^2}{F_1} \quad [\text{Eq 29}]$$

where:

$$F_M = \frac{F_o}{2Q_o} \left[ \left( 1 + \frac{Q_o}{Q_N} \right) (S_M - 1) \right]^{\frac{1}{2}}$$

#### Comparison with Earlier Results

Much has been reported regarding the design of optimum matching networks when the load is complex. In these analyses, the assumption has usually been made that the matching network is made up of lossless elements. In order to compare the results of this investigation with the earlier results, it is necessary to set  $\Delta = 0$ . The comparison may be made by showing the relationship between the normalized bandwidth and  $S_M$ , the maximum SWR over the operating band; see Figure 8.

Equation 6 and Equation 8 provide the required formulas for the dipole matched at resonance and optimum transformer matching, respectively. For the case of transformer/resonator optimum bandwidth matching, from Equation 22,

$$B_{Nmax} \Big|_{\Delta=0} = (S_M^2 - 1)^{\frac{1}{2}} \quad [\text{Eq 30}]$$

For the case of transformer/resonator matching with two perfect match frequencies, from Equation 27,

$$B_N \Big|_{\Delta=0} = [2(S_M - 1)]^{\frac{1}{2}} \quad [\text{Eq 31}]$$

In addition to the relationships derived above, Figure 8 gives the Bode-Fano limiting case, which shows the maximum bandwidth theoretically attainable with an infinite number of elements in the matching network.<sup>10</sup> For this case,

$$B_{Nmax} = \frac{\pi}{\ln \left( \frac{S_M + 1}{S_M - 1} \right)} \quad [\text{Eq 32}]$$

The cases of transformer and transformer/resonator matching for maximum bandwidth exactly coincide with Fano's results for the equivalent situations; his results were achieved using a more general technique, which involved a graphical solution for the final result. (See Note 10.) Incidentally, the terminology of Fano is different than that used in this paper, but the necessary translations were made to prepare Figure 8.

For comparison, the case that shows how much additional bandwidth could be obtained if one more resonator were added to the matching network is also given in Figure 8. This result is derived from Fano and Levy; the two-resonator topology is shown in Figure 9. (See Notes 10 and 11.) It has been shown that the analytically derived matching network optimization is not always optimum.<sup>15, 16</sup> Thus, for the two-resonator case, the curve shown may not be optimum, however, for the lossless matching network cases presented in this paper, it may be shown that the true optimum has indeed been found.

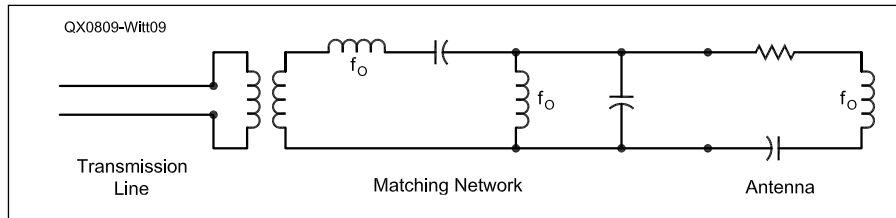


Figure 9 — Transformer/two-resonator matching.

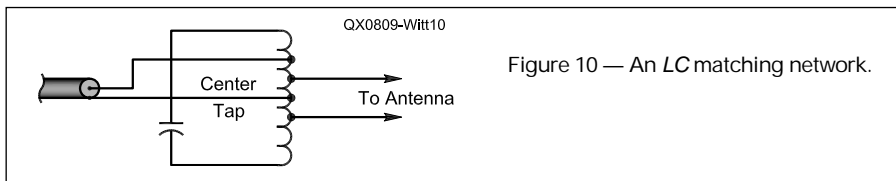


Figure 10 — An LC matching network.

### Practical Matching Networks

Two types of matching networks are presented: an LC network and a transmission line resonator. Each is based on the transformer/single resonator topology of Figure 4A. The lumped LC resonator/transformer exhibits low matching network loss and has the potential for providing the balun function, allowing an unbalanced feed line to drive a balanced antenna without radiation from the feed line. The transmission line resonator may lead to more loss, but has the advantage that it may be integrated with the radiator.

In order to illustrate some of the important practical points associated with the design of a matching network, a specific example will be considered. The antenna to be matched is a half-wave dipole in free space resonant at 4 MHz. The desired operating bandwidth is 500 kHz. The method of moments using the program, MININEC, was used to compute the driving point impedance of an uncompensated version of the antenna to be matched.<sup>17</sup> The effect of the simplifying assumptions made through the use of the antenna model of Figure 1 may thus be seen. In practice, one may estimate  $R_0$  and  $Q_0$ , or better still, build the uncompensated version of the antenna and measure its feed point impedance prior to a final design of the matching network.

In the examples that follow, unless otherwise noted, the designs achieve the minimum SWR over the operating band. Other assumptions are:

- $F_L = 3.758$  MHz
- $F_H = 4.258$  MHz
- $Z_T = 50 \Omega$

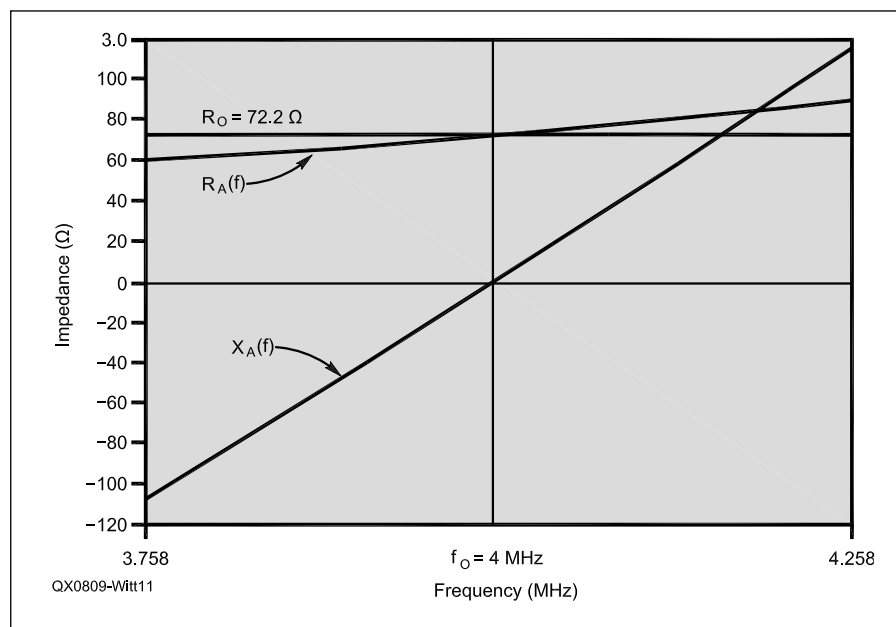


Fig 11 — Impedance versus frequency for a 4 MHz half-wave dipole in free space.

### LC Matching Network

A practical LC matching network is shown in Figure 10. (See Note 2.) The function of a transformer is realized by providing primary and secondary taps on the coil. For the case when a coaxial transmission line is used and the resonant antenna load is balanced, such as a symmetrically-situated center-fed half-wave dipole, the network also serves as a balun. This is accomplished by connecting the shield of the coaxial cable to the center tap of the coil. By connecting the capacitor as shown in the figure, an optimum selection of matching network components may be made. In effect, the inductor is an autotransformer with three functional windings: a primary, a secondary and a capacitor winding.

Figure 11 shows the computed impedance of the 4 MHz half-wave dipole for the case when it is made of no. 14 AWG wire (diameter = 0.064 inch). Forty segments were used in the computer analysis. From these data, the antenna Q and radiation resistance at resonance are determined:

- $Q_0 = 12.2$
- $R_0 = 72.2 \Omega$
- Dipole length = 120.1 feet

By assuming  $Q_N = 300$ , which is a readily attainable value in most practical situations, the following results are obtained:

- $F_0 = 4$  MHz
- $S_{Mmin} = 1.798:1$
- $Z_{Nopt} = 19.41 \Omega$
- $N_Z = 2.58:1$
- $L_{MNE} = 0.176$  dB

After selecting the capacitor, the tapped inductor of Figure 10 may be designed. This procedure will not be covered here. It is important to realize that the components chosen must be capable of withstanding the large electrical stresses encountered when high transmitted power is involved. In the author's experience, high radio frequency currents that flow in the capacitor in this kind of service place particularly high demands on that component.

Figure 12A shows the SWR and matching network loss versus frequency characteristic for this example when the idealized RLC dipole model is assumed. Also shown for comparison is the SWR of a dipole when optimum transformer matching is employed. It is noteworthy from Figure 12B that when the actual dipole impedance frequency dependence is accounted for, the differences in SWR and loss are small. Note also from Figure 12D that the SWR characteristic may be made symmetrical by a slight perturbation of the dipole resonant frequency (-0.15%), and an increase of the resonator natural frequency (+0.6%).

### Transmission Line Resonator Matching Network

Another way to realize a transformer/resonator is to use a resonant length of transmission line. (See Notes 3 and 4.) The simplest form is a transmission line one-quarter wavelength long terminated with a short circuit at one end and an open circuit at the other end. In what follows, this form of resonator will be used, although there are applications where longer transmission line resonators could be used. In these latter cases, the power handling capacity of the matching network would be larger, but the resonator Q would be unchanged from the quarter-wave case.

Figure 13 shows a quarter-wave resonator/transformer.<sup>7,8</sup> By driving and loading the resonator at different points, the function of a transformer is realized. The resonator has a Q that is related to the loss of the transmission line at the resonant frequency:

$$Q = \frac{2.774F_0}{AV} \quad [\text{Eq 33}]$$

where:

A = transmission line attenuation at  $f=F_0$  (dB/100 feet)

V = velocity factor.

It is worth noting that since  $A \propto \sqrt{f}$  for many transmission lines,  $Q_N \propto \sqrt{F_0}$  (approximately). Hence, using the same cable type, higher Q values are obtained at higher frequencies. First order approximations to the equivalent circuit parameters are:

$$Z_N = \frac{4Z_R}{\pi} \sin^2 \theta_S \quad [\text{Eq 34}]$$

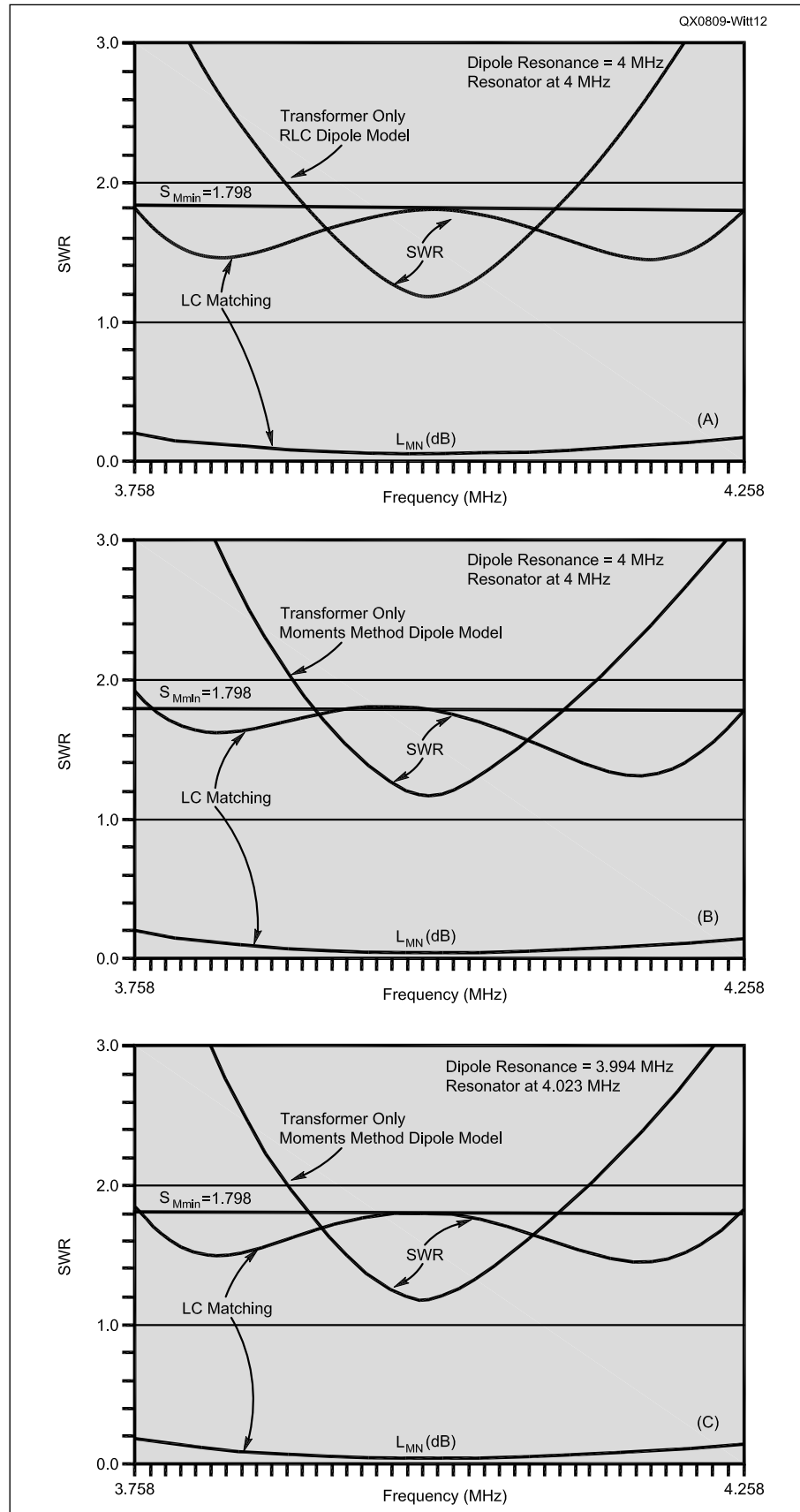


Figure 12 — SWR and matching network loss for LC matching network example. Part A shows the calculated matching network parameters, RLC dipole model. Part B shows the calculated matching network parameters, moments method dipole model. Part C shows the perturbed dipole and matching network resonances, moments method dipole model.

and:

$$N_Z = \frac{\sin^2 \theta_s}{\sin^2 \theta_p} \quad [\text{Eq 35}]$$

where:

$Z_R$  = characteristic impedance of the resonator transmission line (ohms), and  $\theta_s$  and  $\theta_p$  are the electrical angles of the secondary and primary taps, respectively, measured from the shorted end of the resonator.

These approximations are useful only if other significant resonances are well separated from the band of interest. For example, the anti-resonance of the open stub occurs above the operating band and as the secondary tap approaches the short, that frequency approaches the operating band of the antenna system. In most practical cases, however, the equivalent circuit shown provides a sufficiently accurate initial set of matching network parameter values.

The application of the quarter-wave resonator/transformer as a matching network is shown in Figure 14. First the electrical angles (in radians),  $\theta_s$  and  $\theta_p$ , are determined:

$$\theta_s = \sin^{-1} \left( \frac{\pi Z_N}{4Z_R} \right)^{\frac{1}{2}} \quad [\text{Eq 36}]$$

and

$$\theta_p = \sin^{-1} \left( \frac{\pi Z_N}{4Z_R N_Z} \right)^{\frac{1}{2}} \quad [\text{Eq 37}]$$

These results are used to determine the lengths (in feet) of the transmission line segments as defined in Figure 14:

Shorted stub:

$$L_S = \frac{492V\theta_p}{\pi F_O} \quad [\text{Eq 38}]$$

Link:

$$L_L = \frac{492V\theta_s}{\pi F_O} - L_S \quad [\text{Eq 39}]$$

Open stub:

$$L_O = \frac{246V}{F_O} - L_S - L_L \quad [\text{Eq 40}]$$

Incidentally, it may be shown that  $L_S$  is independent of  $R_0$ , a fact that may be used to advantage in a situation when  $R_0$  is not known accurately.

### The Coaxial Resonator Match

By recognizing that the fields and currents in a resonator made from coaxial cable are mostly confined to be within the cable, one can, in effect, integrate the resonator within the antenna radiator. This has been called the coaxial resonator match and is shown in Figure 15 for the case of a half-wave dipole.<sup>3</sup> <sup>4</sup> Note that the elements of the matching network in Figure 14 are contained within the structure. Currents flowing on the inside of the resonator shield are associated with the

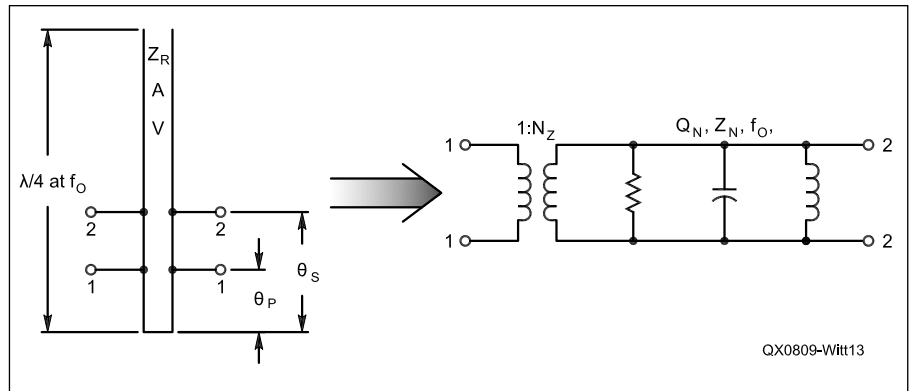


Figure 13 — The quarter-wave resonator/transformer.

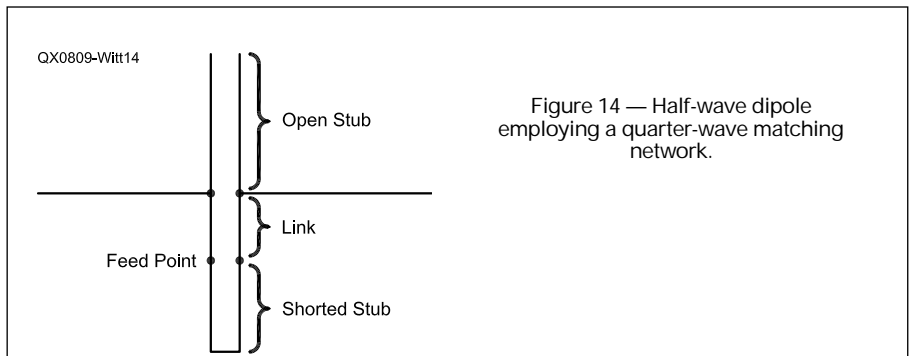


Figure 14 — Half-wave dipole employing a quarter-wave matching network.

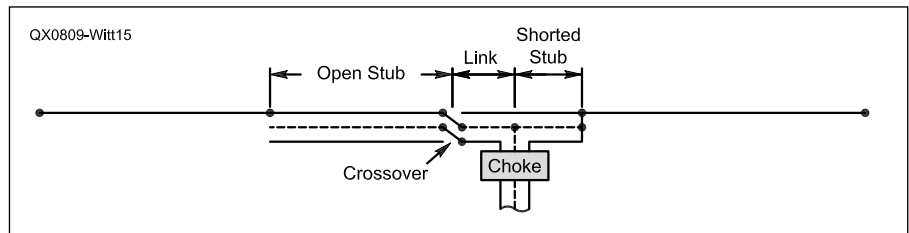


Figure 15 — The coaxial resonator match.

resonator; currents flowing on the outside of the shield are the usual dipole radiator currents. Radiation from the feed line, which is connected off-center for the above design equations to apply, is avoided by the use of a longitudinal choke as seen in the figure. A minor modification of the design procedure would permit the feed line to be connected to the physical center of the antenna but this would not eliminate the desirability of a longitudinal choke when an arbitrary length of feed line is used.

In Figure 15 the extensions necessary to build out the antenna length to one-half wavelength are made from wire. These lengths could be made from the same coaxial cable material as the resonator; the results are similar. Assuming that the entire dipole is made from RG213U coaxial cable (shield diameter = 0.3 inch), the following design input parameters were derived using MININEC:

$$Q_0 = 10.2$$

$R_0 = 72.1 \Omega$   
 Dipole length = 119.5 feet  
 For RG213U cable,  
 $Z_R = 50 \Omega$   
 $A = 0.4 \text{ dB}/100 \text{ feet at } 4 \text{ MHz}$   
 $V = 0.66$   
 Hence,  $Q_N = 42.0$ , leading to the following results:

$F_0 = 4 \text{ MHz}$   
 $S_{\text{Mmin}} = 1.516:1$   
 $Z_{\text{Nopt}} = 17.36 \Omega$   
 $N_Z = 1.99:1$   
 $L_{\text{MNE}} = 1.00 \text{ dB}$   
 $L_S = 9.8 \text{ feet}$   
 $L_L = 4.4 \text{ feet}$   
 $L_O = 26.4 \text{ feet}$

Figure 16A shows the SWR and matching network loss for the case when the RLC dipole model and lumped matching network approximation are used. In Figure 16B, a simulation program that uses the MININEC-derived dipole model and an accurate representation of the transmission line segments

was used to determine the SWR and matching network loss. One observes that the simulation yields results that closely match those predicted from the approximate analysis. An interesting observation is that a degree of serendipitous self-compensation for the imperfections of the lumped element dipole and matching network models takes place when a moments method dipole model and transmission line matching network are used. This is made clear when Figure 12B (where self-compensation is not present) and Figure 16B are compared.

A matter of practical interest is the electrical stress on the coaxial cable in this application. At 4 MHz, the loss in the cable is primarily resistive. To accurately calculate the current and voltage distribution within the resonator, it is necessary to use the complex value of characteristic impedance,  $Z_R$ . The segment lengths associated with Figure 16B were used. When the total power into the antenna plus matching network is one kilowatt, the maximum equivalent power stress (occurring at the low end of the operating band) is 12.5 kW. It is the current in the center conductor that places the highest stress on the cable. The peak voltage at the open circuit occurs at the high end of the operating band. When the total power into the antenna plus matching network is one kilowatt, this voltage is 826 V.

#### Minimizing Matching Network Loss

In many applications, the allowable SWR over the operating band is larger than the minimum achievable SWR,  $S_{Mmin}$ . By designing for this larger SWR, lower matching network loss may be obtained. The matching network loss may be improved by using Equation 20 to find  $Z_N$ . An example will illustrate this point.

In the previous example, the SWR over the 500 kHz operating band was 1.516:1. It will be assumed that the application allows, instead,  $SWR < 2:1$ . Using Equations 10, 15 and 20, the following results are obtained:

$$Z_N = 51.5 \Omega$$

$$N_Z = 2.79:1$$

$L_{MNE} = 0.36$  dB (compared to 1.00 dB for the "optimum" case)

Note that in the use of Equation 20, the "+" root was chosen in order to minimize the loss. This case is shown in Figure 17A when the approximate dipole and lumped element coaxial resonator match models are used. The more accurate dipole and matching network models were used to obtain the results of Figure 17B. By perturbing the dipole resonant frequency and segment lengths, an SWR shape similar to that of Figure 17A is obtained, as shown in Figure 17C.

The electrical stress on the resonator coaxial cable is reduced when match quality is traded for improved matching network

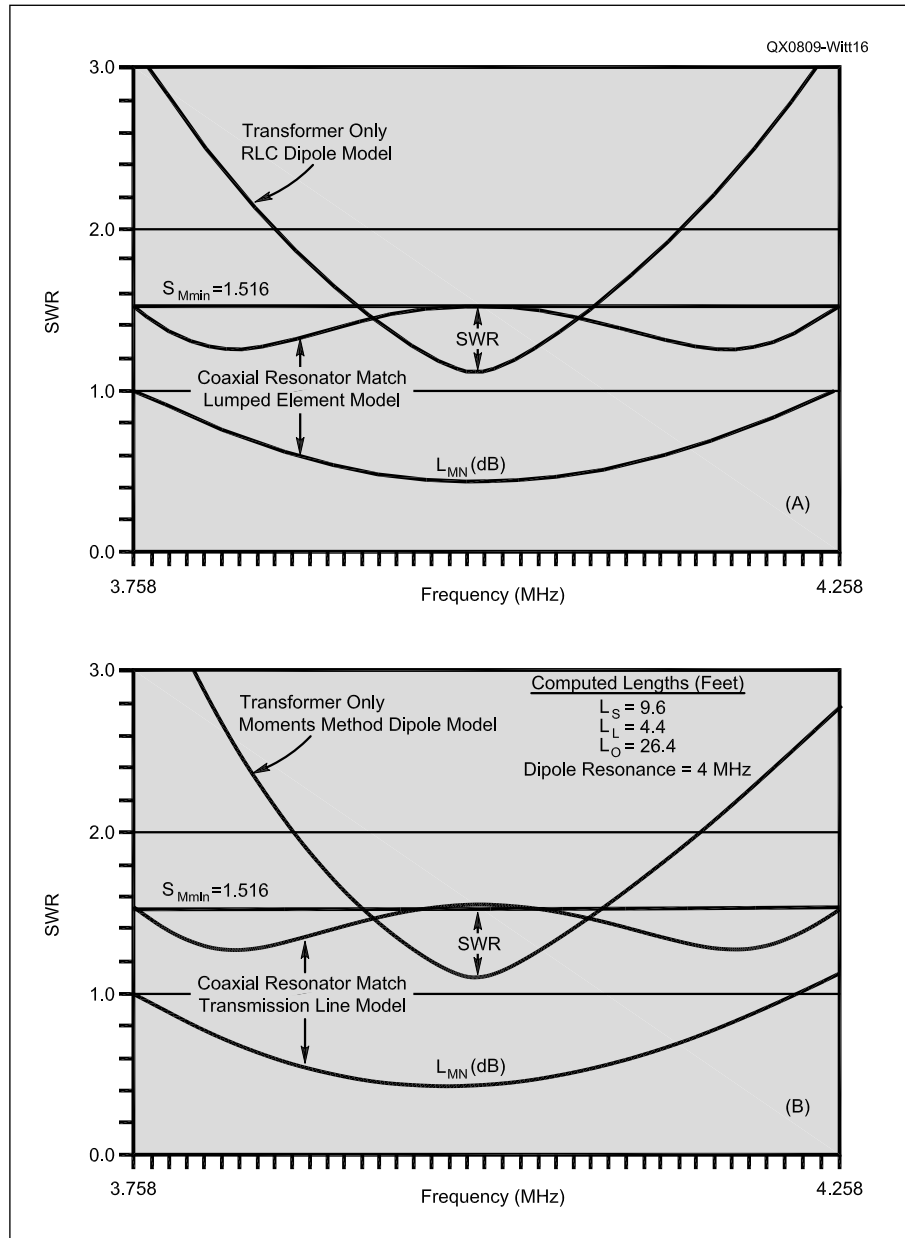


Figure 16 — SWR and matching network loss for coaxial resonator match example. Part A shows the RLC dipole model and lumped element matching network. Part B shows the moments method dipole model and transmission line model of the matching network.

loss. For the case of Figure 17C, when the total power into the antenna plus matching network is one kilowatt, the maximum equivalent power stress is 6.2 kW and the peak voltage at the open is 507 V (compared to 12.5 kW and 826 V, respectively, for the "optimum" case of Figure 16B).

If the "-" root had been chosen, the following parameters would have been obtained:

$$Z_N = 10.2 \Omega$$

$$N_Z = 2.47:1$$

$$L_{MNE} = 1.59$$
 dB

Compare this result, shown in Figure 18, with Figure 17A; the matching network loss is higher by a factor of 4.4. In general, the

solution that yields the highest value of  $Z_N$  will have the lowest loss.

#### Conclusion

An important matching network for resonant antennas has been analyzed in detail. The importance stems from the large match improvement that a simple transformer/resonator matching network provides. The degree of improvement relative to simple transformer matching on one end and higher order matching on the other has been provided. Design equations, which account for the losses in the matching network, have been derived and applied to specific examples.



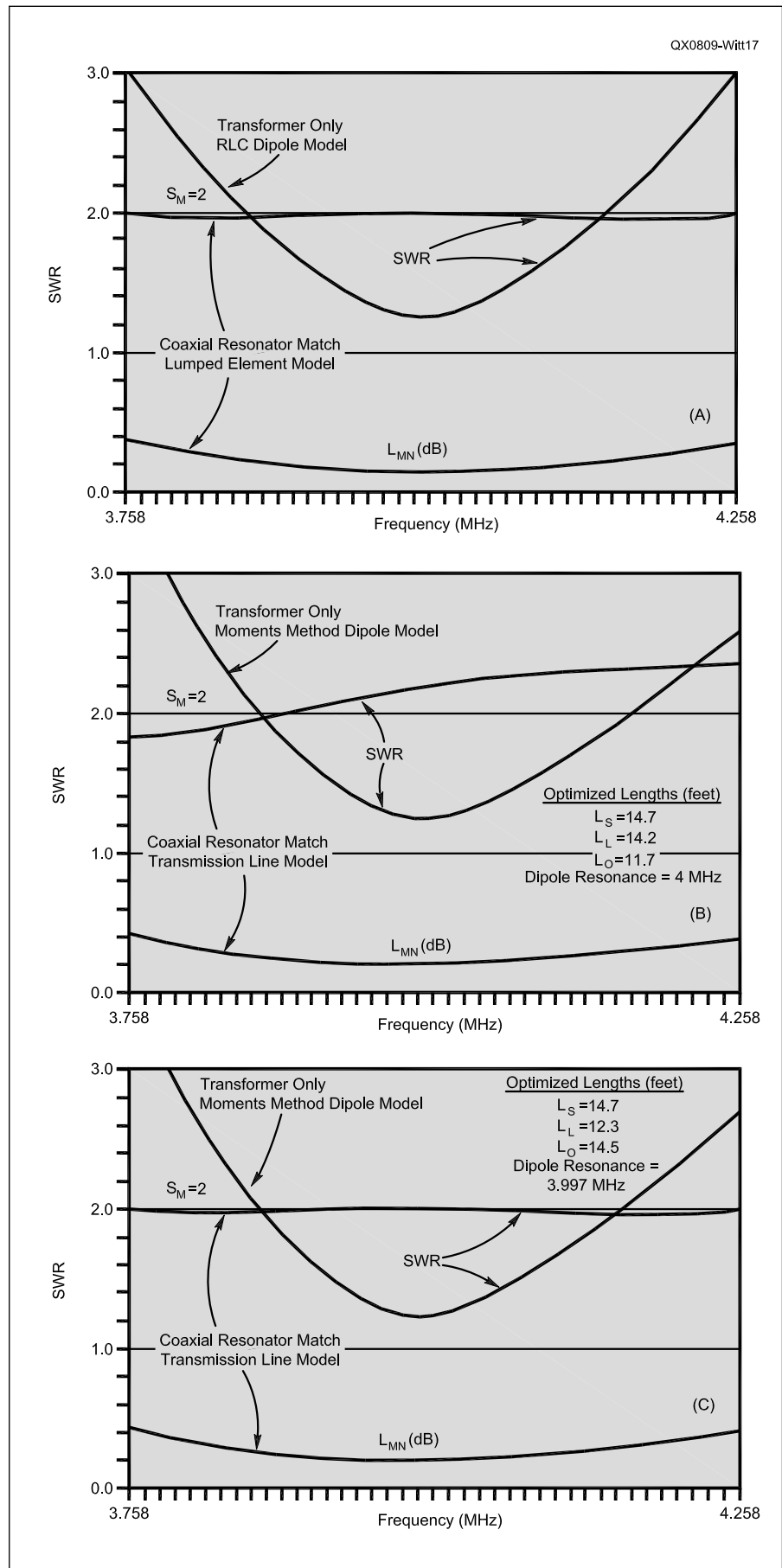
Acknowledgement

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Notes

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Figure 17 — SWR and matching network loss characteristic when match quality is traded for reduced loss. Part A shows the RLC dipole model and lumped element matching network. Part B shows the moments method dipole model and calculated segment lengths. Part C is the same as B, except dipole resonance and segment lengths have been perturbed to restore the SWR characteristic of Part A.



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Frank Witt, AI1H was first licensed in 1948. He holds BS and MS degrees in Electrical Engineering from Johns Hopkins University. He is retired from Bell Telephone Laboratories, where he worked for 37 years. He is a Life Member of the IEEE.

Frank has written about 20 articles related to Amateur Radio. His novel contributions include the top-loaded delta loop, the coaxial resonator match, the transmission line resonator match and the geometric resistance box. He received the 1995 ARRL Technical Excellence Award, and serves as an ARRL Technical Advisor. His other interests include golf and tennis.

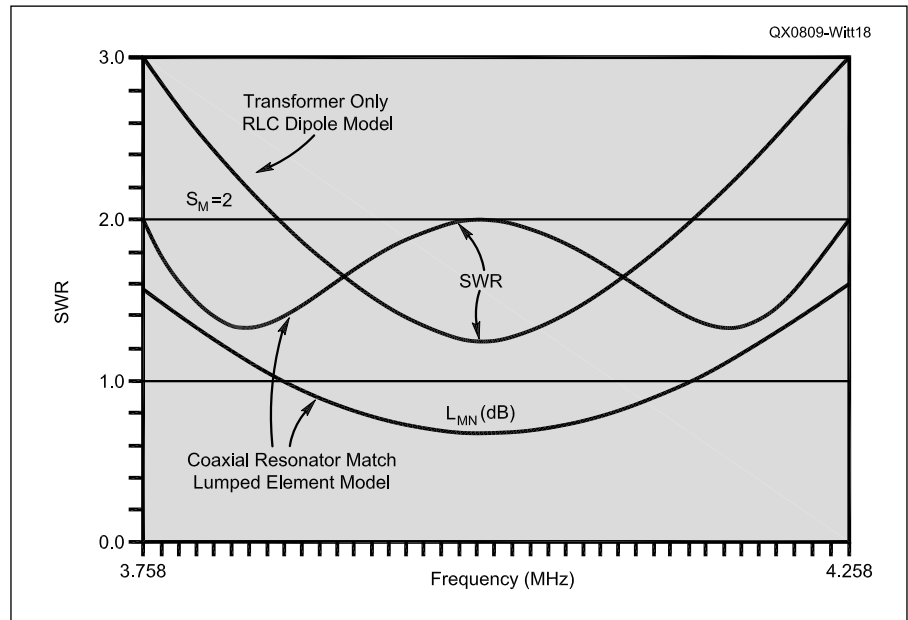




Figure 18 — SWR and matching network loss when the “-” root of Equation 20 is chosen.

QEX



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