Table 1
Maxwell's Equations as Text and (Over)Simplified Math

Equation 1	Gauss's Law: electric charge in some volume, q_V, generates an electric field, *E*.	$\nabla \bullet E = q_V/\varepsilon_0$
Equation 2	There is no independent magnetic "charge" analogous to electric charge.	$\nabla \bullet H/\mu_0=0$
Equation 3	Faraday's Law: a changing magnetic field, *H*, creates an electric field, *E*.	$\nabla \times E=-\mu\ (dH/dt)$
Equation 4	Ampere's law: moving charge (current, *I*) and changing electric fields, *E*, can both create a magnetic field, *H*.	$\nabla \times H=I+\varepsilon(dE/dt)$

these surfaces, avoiding arcs and corona.

Figure 1 is an illustration of gradient on a topographic map, which measures *gravitational potential*, also known as "elevation." The two blue lines, A and B, each represent a horizontal distance of 2,000 feet. Along which line is the gradient of elevation (vertical distance per horizontal distance) the greatest from end to end? The heavy brown contour lines are spaced 100 feet apart, so the net gradient along A from end to end is 200 feet in 2,000 feet or 0.1 feet per foot. B touches six heavy brown lines for a gradient of 600 / 2,000 = 0.3 feet per foot. The gradient symbol in Maxwell's equations includes the gradient in all three dimensions, not just two as in this map.

Divergence (•)

Divergence, represented by the • symbol, can also be illustrated on a topographic map. Divergence describes whether a value, such as gravitational potential (elevation) or electrical potential (voltage), is increasing or decreasing through a curve or across a surface. Start with the contour circuit C, which surrounds the central peak, and imagine a rolling ball as your "gravity-o-meter." What would a ball do if placed on the contour circuit line? Everywhere around circuit C, gravitational potential increases to the "inside" and decreases "outside," so the ball would roll away from the central peak. We would say there is a high positive divergence in gravitational potential (elevation) across the circuit. (If the circuit was drawn around a sinkhole, there would be a high negative divergence.)

The case of the circuit labeled D is not as simple. Part of the circle is on the slope of a nearby peak, two parts are in separate parts of a valley, and some is on the slopes of the central peak. Depending on location, a ball dropped on the circuit would roll toward the interior (1 and 6 o'clock positions, negative divergence), away from the interior (4 o'clock, positive divergence), or along the circuit (10 o'clock, zero divergence). Figuring out whether net divergence was positive or negative would require you to sum it up at each point around the circuit. Mathematically, this is an integration around the whole circuit, and it is shown as an integration symbol with a small circle in the middle (see Rautio's website in Note 3). Like the gradient, divergence in Maxwell's equations includes all three dimensions.

Pencils Down

While I'll still write for *QST* and ARRL, this and the following column are the conclusion of a wonderful 15-year run during which "Hands-On Radio" covered topics from simple component characteristics to transistor and op-amp circuit design, using simulator and design software, and now Maxwell's equations. Your response has been great, and I am indebted to all the experts who pay close attention, keeping me on the rails at times with suggestions and (cough, cough) corrections on occasion. Thank you.

Curl (×)

The final tool in the set is *curl*, and that is something we can't show on a topographic map. Curl is derived from *circulation*, which could be understood as the push from gravity along a closed path such as one of our contour circles. Curl is denoted by the × symbol, which is also used to represent the mathematical *cross product* of two vectors.

Along circuit C in Figure 1, you would get no push anywhere because the gravitational potential at each point (elevation) is the same. Along circuit D, the push might be in one direction then in the other, but around the whole circuit, the net circulation is zero. Otherwise you could go up or down forever like an Escher staircase.

Curl is the amount of circulation per unit of area, and would be experienced as a twisting or turning force. You can experience curl for yourself. Anyone with boating experience has experienced curl when the current vectors at one end of the vessel are stronger (or have a different direction) than at the other. The twisting force shows the curl of the current's vector field across the surface of the water. Whirlpools and hurricanes also illustrate curl.[4]

Gradient, divergence, and curl play a role in our day-to-day radio operating and are illustrated by the online video in Note 5. We'll take a closer look at the equations and find out how they lead to electromagnetic waves in the next column.

Notes

1 All previous "Hands-On Radio" experiments are available to ARRL members at **www.arrl.org/hands-on-radio**.
2 **en.wikipedia.org/wiki/Oliver_Heaviside** and Paul Nahin, *Oliver Heaviside: The Life, Work, and Times of an Electrical Genius of the Victorian Age*, IEEE, 2002.
3 J. Rautio, AJ3K, "The Long Road to Maxwell's Equations," Dec. 2014, *IEEE Spectrum*, pp. 36 – 40, 54 – 56, and **www.microwaves101.com/encyclopedias/maxwell-s-equations**.
4 **earth.nullschool.net**
5 **www.youtube.com/watch?v=qOcFJKQPZfo**